

Scientific report
on the implementation of the project PN-II-ID-PCE-2011-3-0118
in the period January - December 2012

During this period four scientific papers were elaborated, four among them being published or submitted for publication in ISI-ranked journals. The content of these four papers, that cover completely the objectives proposed for the first phase, can be synthesized as follows:

1. L. Ornea, M. Verbistky, V. Vuletescu, *Blow-ups of Locally Conformally Kaehler Manifolds*, International Mathematics Research Notices (2012), doi:10.1093/imrn/rms128.

The locally conformally Kaehler manifolds (LCK for short) are complex manifolds covered by Kaehler manifolds such that the deck transformations of the covering group acts by homotheties. One knew that the blow-up of LCK manifolds along points admit LCK metrics too (Tricerri 1982, Vuletescu 2009). In the present paper we studied the existence of LCK metrics on blow-ups along submanifolds of dimension at least 1 of compact LCK manifolds. We found out that, unlike the Kaehler case, where the blow-up space always admit LCK metrics, the blow-ups of LCK manifolds carry LCK metrics if and only if the blow-up locus is Kaehler, with respect to the induced metric. In particular, the blow-ups of compact Vaisman manifolds do not carry LCK metrics, which shows new examples of compact complex manifolds that do not admit LCK metrics (the observation is important since there cannot be topological obstructions to the existence of LCK metrics). Another important consequence is the fact that twistor spaces do not admit LCK metrics which are not Kaehler. Since the twistor spaces that carry Kaehler metrics are known, this corollary offers new examples, in complex dimension at least 3, of compact complex manifolds that do not admit LCK metrics.

2. G. E. Vîlcu, *Ruled CR-submanifolds of locally conformal Kaehler manifolds*, Journal of Geometry and Physics, 62 (6) (2012), 1366-1372.

The concept of CR-submanifold, introduced by A. Bejancu in the context of Kaehler geometry, was later extended in LCK ambient as well. Such a submanifold admits in natural way some canonical foliations, studied by Chen and Piccinni (1985). One of these foliations, denoted F^\perp and called the real foliation, is given by the totally real distribution involved in the very definition of the CR-submanifold, is known to be completely integrable (Blair and Chen, 1979). In this paper the relation between the geometry of the totally real foliation and the geometry of the submanifold was investigated, with emphasis on the links between the foliation and the LCK structure of the ambient space. In particular, one obtained new necessary conditions for a CR-submanifold of an LCK manifold to be ruled with respect to the totally real foliation, respectively for this foliation to be a Riemannian one.

3. M. Vişinescu, G. E. Vilcu, *Hidden symmetries of Euclideanised Kerr-NUT-(A)dS metrics in certain scaling limits*, Symmetry, Integrability and Geometry: Methods and Applications (SIGMA), 8(2012), 058, 15 pages.

The concept of symmetry is a fundamental one in physics, Noether's theorem giving a correspondence between symmetries and the quantities preserved. For geodesic motions in a space-time, the quantities which are usually conserved are correlated to the isometries that correspond to the Killing vectors. Sometimes a space-time admits higher-order symmetries described by Killing symmetric tensors, the so-called Stackel-Killing (S-K) tensors. These symmetries are also known as *hidden symmetries*, the usual example being the Runge-Lenz vector in the Kepler/Coulomb problem. The corresponding conserved quantities are quadratic or, more generally, polynomial with respect to the momentum. Their existence guarantees the integrability of geodesic motions and is fundamentally linked to the separability of the equation of Hamilton-Jacobi and Klein-Gordon at quantum level. The next simple objects that can be studied in connection to the symmetries of a manifold after the S-K tensors are the Killing-Yano (K-Y) tensors, also known in literature as Killing forms or Yano tensors, known to be correlated to nonstandard super-symmetries. The main goal of this paper was to identify two new Killing forms on Einstein-Sasaki spaces obtained from Kerr-Nut-(A)dS metrics under certain condition in the passing to the limit. This result will allow, via a theorem of Semmelmann (2003) a complete description of the set of all Killing forms on such spaces.

4. C. Gherghe, *Harmonic maps and stability on locally conformally Kaehler manifolds*, submitted.

The harmonic maps are critical points of the energy functional. The study of harmonic maps has numerous applications. There are many interesting results concerning harmonic maps on Kaehler manifolds, yet this kind of applications are relatively less studied on locally conformally Kaehler manifolds. The theory of harmonic maps on manifolds endowed with supplementary geometric structures originates in a paper by Lichnerowicz which showed that a harmonic map between two compact Kaehler manifolds is not only harmonic, but also a minimum (in its homotopy class) of the energy functional. In 1995, Ianuş, Ornea and Vuletescu found a sufficient condition for a holomorphic map from an LCK manifold to a Kaehler one to be harmonic. In the main result of the present paper it is shown that a holomorphic harmonic map from a locally conformally Kaehler manifold to a Kaehler one is weakly-stable. In particular, one reproves, in a different manner, the result in Ianuş-Ornea-Vuletescu. It is also shown that a pluriharmonic map from a locally conformally Kaehler to a Kaehler one is actually constant.

Talks at national and international conferences or in departmental seminars:

G.E. Vîlcu:

Foliated CR-submanifolds at The International Conference on Applied Analysis and Algebra, June 20–24, 2012, Istanbul.

Canonical foliations on Cauchy Riemann submanifolds at The International Conference on Complex Analysis and Related Topics, The 13th Romanian-Finnish Seminar, June 26-30, 2012, Ploiești,

Semi-Riemannian submersions from manifolds endowed metric mixed 3-structures at the 6th European Congress of Mathematics, July 2-7, 2012, Krakow (poster)

C. Gherghe

Aplicații armonice pe varietăți local conform Kähler, A XVI-a Conferință Anuală a Societății de Științe Matematice din România, 19-20 octombrie 2012, Ploiești.

Liviu Ornea

Blow ups and submersions of locally conformally Kaehler manifolds in the Geometry Seminars of the Universities: „La Sapienza” -Rome, Hamburg, Marburg, Koeln, Oberseminar of the „Max Planck” Institute Bonn (all in 2012)

Victor Vuletescu

Number fields in locally conformal geometry, Mathematische Gesellschaft of the University of Göttingen.