SCIENTIFIC REPORT ON THE IMPLEMENTATION OF THE PROJECT PN-II-ID-PCE-2011-3-0118 DURING 1.XII.2013-1.XII.2014

During this period 4 scientific papers were elaborated, all being accepted or published in ISI-ranked journals. The content of these 4 papers, that cover completely the objectives proposed for this period, can be synthesized as follows:

1. P. Gauduchon, A. Moroianu, L. Ornea: *Compact homogeneous LCK manifolds are Vaisman*, Math. Annalen. DOI:10.1007/s00208-014-1103-x, in press.

We show that any compact homogeneous LCK manifold has parallel Lee form, thus being Vaisman. This completely classifies compact homogeneous LCK manifolds.

2. V. Vuletescu, *LCK metrics on Oeljeklaus-Toma manifolds versus Kronecker's theorem*, Bull. Math. Soc. Sci. Math. Roum., Nouv. Ser Tome 57(105), No. 2, 225-231, (2014).

The Oeljeklaus-Toma manifolds were introduced relatively recently (Ann. Inst. Fourier, 2005) being one of the few classes of non-Kahler manifolds of small algebraic dimension know so far. They are compact complex manifolds associated to number fields K with s real embeddings and s complex embeddings and to some subgroups U of the units group of K. Their first application, which appeared already in the original paper referred previously was the disproval of an old conjecture due to I. Vaisman about the Betti number of locally conformally Kahler manifolds (LCK), in the case $t = 1$ and $s = 2$. Also, in the same paper, it was proven that these manifolds carry no LCK metrics for $s = 1$ and arbitrary $t > 1$.

The problem of existence of LCK metrics on these manifolds is still open in the general case. In a recent paper (H. Kasuya, Bull. London Math. Soc, 2012) it was proven, using techniques of cohomology of Lie algebras, that some classes of Oeljeklaus-Toma manifolds do not admit LCK metrics which are also Vaisman (i.e. with parallel Lee form).

In the reported paper it is shown, using techniques coming from algebraic number theory (more precisely, using some results among the many generalisations of Kronecker's units theorem) that in the case $s \geq t$ (and arbitrary U) the Oeljeklaus-Toma manifold do not admit LCK metrics (no only Viasman metrics).

The paper was already cited in the article: *Nonreciprocal units in a number field with an application to Oeljeklaus-Toma manifolds,* A Dubickas - New York J. Math, 2014.

3. B.Y. Chen, G.E Vîlcu, *Geometric classifications of homogeneous production functions*, Applied Mathematics and Computation, Volume 225, December 2013, 345-351.

In the first part of the paper, we generalize a theorem of F. Brickell (J. London Math. Soc. 42 (1967)) concerning surfaces described as graphs of homogeneous functions, proving the following: Let f be a twice differentiable, r-homogeneous, non-constant, real valued function of n variables $(x_1, \dots, x_n)$ on an open domain $D \subset \mathbb{R}^n$, $n \geq 2$. Then the hypersurface of $\mathbb{E}^{n+1}$ defined by

$$z = f(x_1, \dots, x_n), \ \ (x_1, \dots, x_n) \in D,$$

is flat if and only if either f is linearly homogeneous, *i.e.* $r = 1$, or

$$f = (c_1x_1 + c_2x_2 + \cdots + c_nx_n)^r \tag{1}$$

for some real constants $c_1, \dots, c_n$.

În the second part of the paper, the above result is used in the study of the shape and the properties of the production possibility frontier, a subject of great interest in economic analysis. In particular, some classical results of T. Inoue (International Economic Review 25(2) (1984)) and K. Abe, H. Okamoto, M. Tawada (The Canadian Journal of Economics 19 (1986)) are generalized.

4. V. Slesar, M. Visinescu, G.E. Vîlcu, *Special Killing forms on toric Sasaki-Einstein manifolds*, Physica Scripta, Volume 89 (2014), Number 12, 125205 (7 pp).

In this paper we study the interplay between complex coordinates on the Calabi-Yau metric cone and the special Killing forms on the toric Sasaki-Einstein manifold. In the general case we give a procedure to locally construct the special Killing forms. In the final part we exemplify the general scheme in the case of the $5-$dimensional $Y^{p,q}$ spaces.

Talks at national and international conferences or in departmental seminars:

1. L. Ornea: *Homogeneous LCK manifolds*, Laboratory of algebraic geometry and its applications, Faculty of Mathematics, Higher School of Economics, Moscow, 28.02.2014.
2. L. Ornea: *An embeddding theorem in complex geometry* (in Romanian), Romanian Academy, 02.06.2014.
3. C. Gherghe: *Harmonic maps vs Yang-Mills fields*, Real and complex differential geometry, 8-12 September 2014, Bucureşti.
4. G.E Vîlcu, *Bundle-like foliations and submersions in quaternion-like geometries*, Sungkyunkwan University, Suwon, 23 August 2014.

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