Scientific report on the implementation of the project PN-III-P4-ID-PCE-2016-0065 during 2019

During this period 10 scientific papers were elaborated, 2 among them being accepted or published in ISI-ranked journals and 8 papers are under review. Moreover, the article [C. Gherghe: *On a Yang-Mills Type Functional*], submitted for consideration for a possible publication in 2017, was accepted and published in *SIGMA*, 2019, Volume 15, 022, the paper [M. Stanciu: *Locally conformally symplectic reduction*] submitted for evaluation in 2018 was accepted and published in Ann. Glob. Anal. Geom. 56(2) (2019), 245–275, while the paper [L. Ornea, A. Otiman: *A characterization of compact locally conformally hyperkähler manifolds*], submitted for evaluation in 2018, was accepted and published in Annali di Matematica Pura ed Applicata 198(5) (2019), 1541–1549. The content of these 10 papers realized in 2019, that cover completely the objectives proposed for the third phase, can be synthesized as follows.

1. M. Aquib, J.W. Lee, G.E. Vîlcu, D.W. Yoon: *Classification of Casorati ideal Lagrangian submanifolds in complex space forms*, Diff. Geom. Appl. 63 (2019), 30–49.

In this paper, using optimization methods on Riemannian submanifolds, we establish two improved inequalities for generalized normalized δ-Casorati curvatures of Lagrangian submanifolds in complex space forms. We provide examples showing that these inequalities are the best possible and classify all Casorati ideal Lagrangian submanifolds (in the sense of B.-Y. Chen) in a complex space form.

2. G.E. Vîlcu: *Almost product structures on statistical manifolds and para-Kähler-like statistical submersions*, Va apărea în B. Sci. Math.

We investigate statistical manifolds endowed with almost product structures. We prove that the statistical structure of a para-Kähler-like statistical manifold of constant curvature in the Kurose's sense is a Hessian structure. We also derive the main properties of statistical submersions which are compatible with almost product structures. The results are illustrated by several nontrivial examples. In particular, we show that the statistical manifolds corresponding to some standard statistical models, like the well-known normal distribution, Poisson distribution, multinomial distribution, multivariate normal distribution, Dirichlet distribution and Von Mises-Fisher distribution, can be equipped with such structures.

3. M. Stanciu: *Locally conformally symplectic reduction of the cotangent bundle* - preprint 2019.

In a previous article, we introduced a reduction procedure for locally conformally symplectic manifolds at any regular value of the momentum mapping. We use this construction to prove an analogue of a well-known theorem in the symplectic setting about the reduction of cotangent bundles.

4. L. Ornea, M. Verbitsky: *Twisted Dolbeault cohomology of nilpotent Lie algebras* - preprint 2019.

It is well known that cohomology of any non-trivial 1- dimensional local system on a nilmanifold vanishes (this result is due to L. Alaniya). A complex nilmanifold is a quotient of a nilpotent Lie group equipped with a left invariant complex structure by an action of a discrete, co-compact subgroup. We prove a Dolbeault version of Alaniya's theorem, showing that the Dolbeault cohomology of a nilpotent Lie algebra with coefficients in any non-trivial 1-dimensional local system vanishes. Note that the Dolbeault cohomology of the corresponding local system on the manifold is not necessarily zero. This implies that the twisted version of Console-Fino theorem is false (Console-Fino proved that the Dolbeault cohomology of a complex nilmanifold is equal to the Dolbeault cohomology of its Lie algebra). As an application, we give a new proof of a theorem due to H. Sawai, who obtained an

explicit description of LCK nilmanifolds. An LCK structure on a manifold M is a Kaeahler structure on its cover $\tilde{M}$ such that the deck transform map acts on $\tilde{M}$ by homotheties. We show that any complex nilmanifold admitting an LCK structure is Vaisman, and is obtained as a compact quotient of the product of a Heisenberg group and the real line.

5. F. Beşleagă, S. Dăscălescu: *Structural matrix algebras, generalized flags and gradings* - preprint 2019.

Flags and flag varieties play a key role in Algebraic Geometry, Representation Theory, Algebraic Groups and Combinatorics. We consider a more general concept of flag and we give some applications. This new kind of flag arises as follows.

Let k be a field. A structural matrix algebra is a subalgebra of a full matrix algebra over k, consisting of all matrices with zero entries on certain prescribed positions, and allowing anything on the remaining positions. A structural matrix subalgebra A of $M_n(k)$ is associated with a preorder relation ρ on the set $\{1, \ldots, n\}$; A consists of all matrices $(a_{ij})_{1 \leq i,j \leq n}$ such that $a_{ij} = 0$ whenever $(i, j) \notin \rho$. We denote $A = M(\rho, k)$.

A general problem in Ring Theory is to describe and classify all group gradings on various matrix algebras. The aim of our paper is to construct and classify a certain class of gradings on structural matrix algebras.

We show that a structural matrix algebra $M(\rho, k)$ is isomorphic to the endomorphism algebra of a certain algebraic-combinatorial structure $\mathcal{F}$, which we call a ρ-flag. If $\mathcal{F}$ is additionally equipped with a G-grading, where G is a group, then its endomorphism algebra $\mathrm{End}(\mathcal{F})$ gets an induced G-graded algebra structure; we denote by $\mathrm{END}(\mathcal{F})$ the obtained G-graded algebra. This grading transfers to a G-grading on $M(\rho, k)$ via the isomorphism mentioned above.

In order to classify G-gradings on the structural matrix algebra $M(\rho, k)$, we first look at the isomorphisms between the algebras $\mathrm{End}(\mathcal{F})$ and $\mathrm{End}(\mathcal{F}')$, where $\mathcal{F}$ and $\mathcal{F}'$ are ρ-flags under the vector spaces V and V'. In particular, if $\mathcal{F}' = \mathcal{F}$, the automorphism group of $\mathrm{End}(\mathcal{F})$ is described as a factor group of a double semidirect product.

For classifying G-gradings arising from graded flags, we consider two G-graded ρ-flags $\mathcal{F}$ and $\mathcal{F}'$, and we look at the isomorphisms between the graded algebras $\mathrm{END}(\mathcal{F})$ and $\mathrm{END}(\mathcal{F}')$. We obtain that $\mathrm{END}(\mathcal{F}) \simeq \mathrm{END}(\mathcal{F}')$ if and only if the connected components of $\mathcal{F}$ and $\mathcal{F}'$ are pairwise isomorphic up to a permutation, some graded shifts and an automorphism of $\mathcal{C}$; here $\mathcal{C}$ is the poset of equivalence classes with respect to the equivalence relation induced by ρ. Using this result, we show that the isomorphism types of graded algebras of the form $\mathrm{END}(\mathcal{F})$ are classified by the orbits of the action of a certain group, which is a double semidirect product of a Young subgroup of S_n, a certain subgroup of automorphisms of $\mathcal{C}$, and G^q, where q is the number of connected components of $\mathcal{C}$, on the set G^n.

6. N. Istrati, A. Otiman, M. Pontecorvo: *On a class of Kato manifolds* - preprint 2019.

We revisit Brunella's proof of the fact that Kato surfaces admit locally conformally Kähler metrics, and we show that it holds for a large class of higher dimensional complex manifolds containing a global spherical shell. On the other hand, we construct manifolds containing a global spherical shell which admit no locally conformally Kähler metric. We consider a specific class of these manifolds, which can be seen as a higher dimensional analogue of Inoue-Hirzebruch surfaces, and study several of their analytical properties. In particular, we give new examples, in any complex dimension $n \geq 3$, of compact non-exact locally conformally Kähler manifolds with algebraic dimension $n - 2$, algebraic reduction bimeromorphic to $\mathbb{CP}^{n-2}$ and admitting non-trivial holomorphic vector fields.

7. D. Angella, N. Istrati, A. Otiman, N. Tardini: *Variational problems in conformal geometry* - preprint 2019.

We study the Euler-Lagrange equation for several natural functionals defined on a conformal class of almost Hermitian metrics, whose expression involves the Lee form θ of the metric. We show that the Gauduchon metrics are the unique extremal metrics of the functional corresponding to the norm of the codifferential of the Lee form. We prove that on compact complex surfaces, in every conformal class there exists a unique metric, up to multiplication by a constant, which is extremal for the functional given by the L^2-norm of $dJ\theta$, where J denotes the complex structure. These extremal metrics are not

the Gauduchon metrics in general, hence we extend their definition to any dimension and show that they give unique representatives, up to constant multiples, of any conformal class of almost Hermitian metrics.

8. V. Slesar, M. Visinescu, G.E. Vîlcu: *Transverse Kähler-Ricci flow and deformations of the metric on the Sasaki space* $T^{1,1}$ - preprint 2019.

We investigate the possibility to obtain locally new Sasaki-Einstein metrics on the space $T^{1,1}$ considering a deformation of the standard metric tensor field. We show that from the geometric point of view this deformation leaves transverse and the leafwise metric intact, but changes the orthogonal complement of the Reeb vector field using a particular basic function. In particular, the family of metric obtained using this method can be regarded as solutions of the equation associated to the Sasaki-Ricci flow on the underlying manifold.

9. L. Ornea, P.-A. Nagy: *Conformal foliations, Kähler twists and the Weinstein construction* - preprint 2019.

We classify both local and global Kähler structures admitting totally geodesic homothetic foliations with complex leaves. The main building blocks are related to Swann's twists and are obtained by applying Weinstein's method of constructing symplectic bundles to Kähler data. As a byproduct we obtain new classes of: holomorphic harmonic morphisms with fibres of arbitrary dimension from compact Kähler manifolds; non-Kähler balanced metrics conformal to Kähler ones (but compatiblewith different complex structures). Some classes of non-Einstein constant scalar curvature Kähler metrics are also obtained in this way.

10. L. Ornea, M. Verbitsky: *Supersymmetry and Hodge theory on Sasakian and Vaisman manifolds* - preprint 2019.

Sasakian manifolds are odd-dimensional counterparts of Kähler manifolds. They can be defined as contact manifolds equipped with an invariant Kähler structure on their symplectic cone. The quotient of this cone by the homothety action is a complex manifold called Vaisman. We study harmonic forms and Hodge decomposition on Vaisman and Sasakian manifolds. We construct a Lie superalgebra associated to a Sasakian manifold in the same way as the Kähler supersymmetry algebra is associated to a Kähler manifold. We use this construction to produce a self-contained, coordinate-free proof of the results by Tachibana, Kashiwada and Sato on the decomposition of harmonic forms and cohomology of Sasakian and Vaisman manifolds. In the last section, we compute the supersymmetry algebra of Sasakian manifolds explicitly.

It is noteworthy that the dissemination of results was performed not only by publishing articles, but also by talks at international conferences and in departmental seminars:

1. L. Ornea: *The Kaehler geometry of the Weinstein construction*, plenary talk at "Bucharest conference on geometry and physics", 2–6.09.2019.
2. L. Ornea: *The Kaehler geometry of the Weinstein construction*, invited talk at Higher School of Economics, Moscow (Russia), September 27, 2019.
3. V. Vuletescu: *Locally conformally Kähler surfaces: a survey*, "Max LX - Miniworkshop on Non-Kählerian Geometry", Florenta (Italia), 27–28 May 2019.
4. V. Vuletescu: *On the analytic structure of LCK 3-folds with positive algebraic dimension*, Congresul Matematicienilor Români, Galaţi, 28 June–3 July 2019.
5. M. Stanciu: *Locally conformally symplectic cotangent bundle reduction*, Workshop on Riemannian and Kähler geometry, Bucureşti, April 15–19, 2019.
6. M. Stanciu: *Locally conformally symplectic reduction*, Workshop for Young Researchers in Mathematics, Bucureşti, 3–4 June 2019.

7. G.E. Vîlcu: *Killing forms on manifolds endowed with remarkable geometric structures*, 2nd JNMP Conference on Nonlinear Mathematical Physics 2019, Santiago, Chile, 26 May – 4 June 2019.

8. G.E. Vîlcu: *Twistor forms on manifolds endowed with remarkable geometric structures*, The 14th International Workshop on Differential Geometry and Applications, Ploieşti, 9–11 July 2019.

9. F. Belgun: *Torus actions and leaf counting on locally conformally symplectic manifolds*, Workshop on Riemannian and Kähler geometry, Bucureşti, April 15–19, 2019.

10. G.E. Vîlcu: *Geometric properties of production models*, International Conference on Applied and Pure Mathematics, October 31 – November 3, 2019, Iaşi (Romania).

11. G.E. Vîlcu: *On the classification of production models and their associated graph hypersurfaces*, International Workshop on Geometry of Submanifolds, 7-9 November 2019, Istanbul Center for Mathematical Sciences (IMBM), Istanbul (Turkey).

12. A. Otiman: *On a class of Kato manifolds*, Bridging the Gap between Kaehler and non-Kaehler geometry Conference, October 27 - November 1, 2019, Banff (Canada).

13. A. Otiman: *Dolbeault cohomology of Cousin groups and Oeljeklaus-Toma manifolds*, invited talk at Differential Geometry Seminar, Ludwig Maximilian University of Munich (Germany), December 2019.

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