Scientific report on the implementation of the project PN-III-P4-ID-PCE-2016-0065 during 2018

During this period 12 scientific papers were elaborated, 7 among them being accepted or published in ISI-ranked journals and and 5 papers are under review. Moreover, the article [N. Istrati, A. Otiman: *De Rham and twisted cohomology of Oeljeklaus-Toma manifolds*], submitted for consideration for a possible publication in 2017, was accepted for publication in the journal *Ann. Inst. Fourier*. The content of these 12 papers realized in 2018, that cover completely the objectives proposed for the second phase, can be synthesized as follows.

1. A. Moroianu, S. Moroianu, L. Ornea: *Locally conformally Kaehler manifolds with holomorphic Lee field*, Diff. Geom. Appl. 60 (2018), 33–38.

We prove that a compact LCK manifold with holomorphic Lee vector field is Vaisman provided that either the Lee field has constant norm or the metric is Gauduchon (i.e., the Lee field is divergence-free). We also give examples of compact lcK manifolds with holomorphic Lee vector field which are not Vaisman.

2. F. Belgun, O. Goertsches, D. Petrecca: *Locally conformally symplectic convexity*, J. Geom. Phys. 135 (2019), 235-252.

We investigate special lcs and twisted Hamiltonian torus actions on strict lcs manifolds and characterize them geometrically in terms of the minimal presentation. We prove a convexity theorem for the corresponding twisted moment map, establishing thus an analog of the symplectic convexity theorem of Atiyah and Guillemin-Sternberg. We also prove similar results for the symplectic moment map (defined on the minimal presentation) whose image is then a convex cone. In the special case of a compact toric Vaisman manifold, we obtain a structure theorem.

3. G.E. Vîlcu: *An optimal inequality for Lagrangian submanifolds in complex space forms involving Casorati curvature*, J. Math. Anal. Appl. 2018, 465, 1209–1222.

In this paper, we establish an optimal inequality involving normalized δ-Casorati curvature $\delta_C(n-1)$ of Lagrangian submanifolds in n-dimensional complex space forms. We derive a very singular and unexpected result: the lower bounds of the normalized δ-Casorati curvatures $\delta_C(n-1)$ and $\widehat{\delta_C}(n-1)$ in terms of dimension, the holomorphic sectional curvature, the normalized scalar curvature and the squared mean curvature of the submanifold, are different, in contrast to all previous results obtained for several classes of submanifolds in many ambient spaces. We also investigate the equality case of the inequality and prove that a Casorati $\delta_C(n-1)$-ideal Lagrangian submanifold of a complex space form without totally geodesic points is an H-umbilical Lagrangian submanifold of ratio 4. Some examples are discussed in the last part of the paper, showing that the constants in the inequality obtained in this work are the best possible.

4. L. Ornea, M. Verbitsky, V. Vuletescu: *Flat affine subvarieties in Oeljeklaus-Toma manifolds*, To appear in Mathematische Zeitschrift. https://doi.org/10.1007/s00209-018-2121-2

The Oeljeklaus-Toma (OT-) manifolds are compact, complex, non-Kähler manifolds generalizing the Inoue surfaces. We prove that any complex subvariety of smallest possible positive dimension in an OT manifold is also flat affine. This is used to show that generically an OT manifold contains no proper analytic subvarieties.

5. L. Ornea, M. Verbitsky: *Positivity of LCK potential*, To appear in: Journal of Geometric Analysis. https://doi.org/10.1007/s12220-018-0046-y

A locally conformally Kähler (LCK) manifold is a complex manifold, with a Kähler structure on its covering $\tilde{M}$, with the deck transform group acting on $\tilde{M}$ by holomorphic homotheties. One could think of an LCK manifold as of a complex manifold with a Kähler form taking values in a local system L, called the conformal weight bundle, numit fibratul ponderilor conforme. Locally, any LCK form is expressed as an L-valued pluri-Laplacian of a function called LCK potential. We consider a manifold M with an LCK form admitting an LCK potential (globally on M), and prove that M admits a positive LCK potential, too.

6. F. Beşleagă, S. Dăscălescu, L. Van Wyk: *Classifying good gradings on structural matrix algebras*, To appear in: Linear and Multilinear Algebra. https://doi.org/10.1080/03081087.2018.1476447

Let k be a field. If n is a positive integer, an $n \times n$ structural matrix algebra over k is a subalgebra of the full matrix algebra $M_n(k)$ consisting of all matrices with zero entries on certain prescribed positions. Upper triangular matrix algebras, or more generally, upper block triangular matrix algebras are well-known examples. A structural matrix algebra is associated with a preorder relation ρ on the set $\{1, \ldots, n\}$. The aim of the paper is to classify the good G-gradings on the structural matrix algebra $M(\rho, k)$ in the case where ρ is a partial order relation, and G is an arbitrary group. A grading is called good if any matrix unit lying in $M(\rho, k)$ is a homogeneous element. The isomorphism classes of such gradings are proved to be in bijection with the orbits of the action of the automorphism group of ρ on a certain set of functions. We explicitly count the number of isomorphism types of good gradings in some cases where G is finite and the graph associated with ρ has a cyclic associated undirected graph.

7. S. Dăscălescu, C. Năstăsescu, L. Năstăsescu: *Hopf algebra actions and transfer of Frobenius and symmetric properties*, To appear in: Mathematica Scandinavica.

Let H be a finite dimensional Hopf algebra acting on a finite dimensional algebra A. Let A^H be the associated subalgebra of invariants, and let $A\#H$ be the smash product constructed from this action. The aim of the paper is to investigate the transfer of the Frobenius and symmetric properties through the algebra extensions $A^H \subset A \subset A\#H$. This problem is of interest since Frobenius algebras and symmetric algebras occur in representation theory of groups, in quantum group theory, in the theory of compact oriented manifolds, in topological quantum field theory, etc.

8. L. Ornea, A. Otiman: *A characterization of compact locally conformally hyperkaehler manifolds* - preprint 2018.

We give an equivalent definition of compact locally conformally hyperkähler manifolds in terms of the existence of a nondegenerate complex two-form with natural properties. This is a conformal analogue of Beauville's stating that a compact Kähler manifold admitting a holomorphic symplectic form is hyperkähler.

9. L. Ornea, M. Verbitsky, V. Vuletescu: *Classification of non-Kähler surfaces and locally conformally Kähler geometry* - preprint 2018.

We give a mostly self-contained proof of the classification of non-Kähler surfaces based on Buchdahl-Lamari theorem. We also prove that all non-Kahler surfaces which are not of class VII are locally conformally Kahler.

10. A. Otiman, M.Toma: *Hodge decomposition for Cousin groups and for Oeljeklaus-Toma manifolds* - preprint 2018.

We compute the Dolbeault cohomology of certain domains contained in Cousin groups defined by lattices which satisfy a strong dispersiveness condition. As a consequence we obtain a description of the Dolbeault cohomology of Oeljeklaus-Toma manifolds and in particular the fact that the Hodge decomposition holds for their cohomology.

11. M. Stanciu: *Locally conformally symplectic reduction* - preprint 2018.

We present a reduction procedure for locally conformally symplectic (LCS) manifolds with an action of a Lie group preserving the conformal structure, with respect to any regular value of the momentum mapping. Under certain conditions, this reduction is compatible with the existence of

a locally conformally Kähler structure. As a special consequence, we obtain a compatible contact reduction with respect to any regular value of the contact momentum mapping.

12. O. Braunling, V. Vuletescu: *Automorphisms of OT manifolds and ray class numbers* - preprint 2018.

Oeljeklaus-Toma manifods (OT, for short) are a newly introduced class of compact complex manifolds associated to some number fields. We compute the automorphism group of OT manifolds of simple type. We show that the graded pieces under a natural filtration are related to a certain ray class group of the underlying number field. This does not solve the open question whether the geometry of the OT manifold sees the class number directly, but brings us a lot closer to a possible solution.

It is noteworthy that the dissemination of results was performed not only by publishing articles, but also by talks at international conferences and in departmental seminars:

1. L. Ornea: *Positive LCK potentials*, University of Houston, February 9, 2018.
2. L. Ornea: *What is non-euclidian geometry*, University of Houston, February 12, 2018.
3. L. Ornea: *Positive LCK potentials*, Texas Tech University, February 14, 2018.
4. L. Ornea: *The embedding problem in differential geometry*, Texas Tech University, February 15, 2018.
5. L. Ornea: *An introduction to LCK geometry*, course at Universita degli Studi di Firenze, April 2-14, 2018.
6. L. Ornea: *A characterization of locally conformally hyperkähler manifolds*, IMPA-Instituto de Matemática Pura e Aplicada, Rio de Janeiro, June 20, 2018.
7. L. Ornea: *Subvarieties in Oeljeklaus-Toma manifolds*, Universidade Federal Fluminense, Rio de Janeiro, June 22, 2018.
8. L. Ornea: *Subvarieties in Oeljeklaus-Toma manifolds*, Workshop: Special Metrics and Symmetries on Complex Manifold, Regensburg, September 10-15, 2018.
9. A. Otiman: *Cohomology of Oeljeklaus-Toma manifolds*, Complex Geometry and Lie Groups Workshop, Florence, Italy, June 11-15, 2018.
10. A. Otiman: *Cohomology of Oeljeklaus-Toma manifolds*, Oberseminar Geometrie, Topologie und Analysis, Universität zu Köln, Germany, May 4th, 2018
11. A. Otiman: *Cohomology of Oeljeklaus-Toma manifolds*, Séminaire de topologie, géométrie et algèbre, Université de Nantes, France, March 29th, 2018
12. A. Otiman: *Locally conformally Kähler manifolds and their cohomology*, Oberseminar Differential Geometry, Max Planck Institut für Mathematik, Bonn, Germany, February 1st, 2018.
13. M. Stanciu: *Locally conformally symplectic reduction*, SMI course "Conformal geometry, Cartan connection and locally conformally Kaehler structures", Cortona, Italy, 13-19 May, 2018.
14. M. Stanciu: *Locally conformally symplectic reduction*, Workshop "Special Metrics and Symmetries on Complex Manifolds", Regensburg, Germany, 10-15 September, 2018.
15. M. Stanciu: *Locally conformally symplectic reduction*, Workshop Geometry and PDEs, Timişoara, 12-13 October, 2018.
16. V. Vuletescu: *Dolbeault cohomology of some LCK manifolds*, Conference: Cohomology of Complex Manifolds and Special Structures, Levico Terme (Trento), Italy, June 19-22, 2018.

17. G.E. Vîlcu: *Special classes of submanifolds in quaternionic-like geometries*, Gyeongsang National University, Jinju, South Korea, June 18, 2018.

18. G.E. Vîlcu: *Geometric inequalities involving Casorati curvatures*, Gyeongsang National University, Jinju, South Korea, June 19, 2018.

19. G.E. Vîlcu: *Interactions between differential geometry and economic analysis*, International Conference of the Honam-Youngnam Mathematical Societies, June 21-24, 2018, Jeju Island, South Korea.

20. F. Belgun: *The Nash embedding in conformal geometry and the Hill equation*, IMAR monthly lecture, February 21, 2018.

21. F. Belgun: *Convexity for locally conformally symplectic manifolds*, Complex Geometry and Lie Groups Workshop, Florence, Italy, June 11-15, 2018.

22. F. Belgun: *Torus actions on locally conformally symplectic manifolds*, Workshop New trends and open problems in Geometry and Global Analysis, Marburg, Germany, 25.08-2.09.2018.

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